Part-1

* ARBITRAGE PRICING Theory :- (APT)

CMT → leads to → CAPM
APT → leads to → APM : Arbitrage Pricing Model.

P_0 or IV_0 : Present value of future cashflows discounted at Required Rate of Return (R_e).

CAPM → CMT
APM (sum aya h exam me) → APT (aaj tak exam me nhi)

- CAPM :
  $R_f = 5\%$
  Stock - Risky → SR and ✓ ; UR ⊗
  SR ko capture krne k liye kya kre ?
  Risk factor (systematic) → Market portfolio ↓ $E(R_m) = 12\%$ (suppose)
  ∴ Market Risk Premium (MRP) = $R_m - R_f = 7\%$ for 1 Beta
  $R_e = 5 + 7\beta$

- APM : Does market portfolio exist ? - No
  We take proxies → collapse of CAPM --- Rise of APM.
  APM → multiple factors.
  CAPM → single factor → market index.
  CAPM is a single factor APM.

APM unlike CAPM is a multi factor model which considers factors such as GDP growth rate, oil price, interest rate, etc. as systematic risk factors.

However, the no. of risk factors are not specified, so we may have a one factor APM (like CAPM), 2 factor APM, 3 factor APM & so on...

Factors	→ Factor Risk Premium	→ Factor sensitivity
F_1 (market)	FRP_1 or λ_1	β_1
F_2 (growth rate)	FRP_2 or λ_2	β_2
F_3 (oil)	FRP_3 or λ_3	β_3

Factor Risk premium (λ) is not stock specific.... Factor sensitivity (β) is stock specific....

CAPM: $Re = Rf + (Rm - Rf)\beta$

wheree, $Rm - Rf$ = market risk premium.

APM: $Re = Rf + FRP_1 \cdot \beta_1 + FRP_2 \cdot \beta_2 + \cdots + FRP_n \cdot \beta_n$

or $Re = Rf + \lambda_1 \beta_1 + \lambda_2 \beta_2 + \lambda_3 \beta_3 + \cdots + \lambda_n \cdot \beta_n$

(icai writes λ)

Betas → stock dependent.

FRP & Rf → not stock dependent.

Q.46	pg
cw. 61	

As per APM -

$Re = Rf + \lambda_1 \beta_1 + \lambda_2 \beta_2 + \lambda_3 \beta_3$

$= 8 + 4 \times 1.3 + 1 \times 0.3 + (-4) \times 0.2$

$= 12.7\%$

Self practice: HW. Soln. pg.133 - Extra Q.2]

Q.45	pg
cw. 60	

Imp

Equation of APM:

$Re = E(R) = 4.5 + 6.85\beta_1 - 3.5\beta_2 + 0.65\beta_3$

where, $Rf = 4.5$

$\lambda_1 = 6.85$ = FRP of market

$\lambda_2 = -3.5$ = FRP of book price

$\lambda_3 = 0.65$ = FRP of inflation

(i) Expected Return on market index -
(Find 3 factor Betas using weighted average).

factor → market index h

$\beta 1 = 0.25 \times 0.8 + 0.1 \times 0.9 + 0.5 \times 1.165 + 0.15 \times 0.85 = 1$

$\beta 2 = 0.25 \times 1.39 + 0.1 \times 0.75 + 0.5 \times 2.75 + 0.15 \times 2.05 = 2.105$

$\beta 3 = 0.25 \times 1.35 + 0.1 \times 1.25 + 0.5 \times 8.65 + 0.15 \times 6.75 = 5.8$

∴ E(R) for market index

$= 4.5 + 6.85 \times 1 - 3.5 \times 2.105 + 0.65 \times 5.8$

$= 7.7525\%$

Hint : Alternate soln.... exam ⊗
Compute E(R) of all 4 category of stocks then do weighted average

(ii) CAPM → expected return on market index -

$E(R) = R_f + (R_m - R_f)\beta$

$= 4.5 + 6.85 \times 1$

$= 11.35\%$

(iii) Let 'w' be the weight of small cap value stock.
∴ '(1-w)' will be the weight of large cap growth stock.

Target Beta = 1

∴ $w \times 0.9 + (1-w)\, 1.165 = 1$

∴ $w = 62.26\%$

& $1 - w = 37.74\%$

Self practice : Extra Q.1] HW. soln. 131

In CW. Q.46] & 45], FRPs were given and we had to find E(R) ..... what if E(R) of stocks along with Rf is given & you have to find out FRP ? (see Q.26] CW. CAPM)
So what !!! yeh to simultaneous eqn solving h .....

Q.47	Pg
CW. 61	

(iii) Two factor APM :

$E(R) - R_f = \lambda_1 \cdot \beta_1 + \lambda_2 \cdot \beta_2$

A : $15 - 10 = \lambda_1\, 0.8 + \lambda_2\, 0.6$ .... ①

B : $20 - 10 = \lambda_1\, 1.5 + \lambda_2\, 1.2$ .... ②

Multiply eqⁿ ① by 1.5 & Multiply eqⁿ ② by 0.8

$7.5 = 1.2\lambda_1 + 0.9\lambda_2$ .... ③

$8 = \lambda_1 1.2 + 0.96\lambda_2$ .... ④

Now, subtract eqⁿ ③ from ④ -

$0.5 = 0.06\lambda_2$

$\therefore \lambda_2 = 8.33\%$

(i) Own funds = 100000
(+) Funds from short selling B = 50000
Investment in security A = 150000

$W_A = \frac{150000}{100000} = 1.5$

$W_B = \frac{-50000}{100000} = -0.5$

β_1 of the portfolio = $1.5 \times 0.8 + (-0.5) \times 1.5 = 0.45$

β_2 of the portfolio = $1.5 \times 0.6 + (-0.5) \times 1.2 = 0.30$

Imp

(ii) Own funds = 100000
Imp
(+) Borrow @ Rf = 100000
200000

A (1.5) : B (-0.5)

300000 ← Investment → -100000

$W_A = \frac{300000}{100000} = 3$ $W_B = \frac{-100000}{100000} = -1$

$W_{Rf} = \frac{-100000}{100000} = -1$

β_1 of the portfolio = $3 \times 0.8 - 1 \times 1.5 - 1 \times 0 = 0.90$

β_2 of the portfolio = $3 \times 0.6 - 1 \times 1.2 - 1 \times 0 = 0.60$

Self practice: HW. Q.22] pg. 38

Multi-factor Macro-economic Model :-

Step 1 : APM

$$E(R_i) = R_f + \lambda_1 \beta_1 + \lambda_2 \beta_2 + \lambda_3 \beta_3 + \cdots + \lambda_k \beta_k$$

Step 2 : Actual Return

$$A(R_i) = E(R_i) + \beta_1 \cdot Sh_1 + \beta_2 Sh_2 + \cdots + \beta_k Sh_k + e_i$$

where, Sh_k = shock = Actual - forecast for factor k.

However, a wrong Q. was set by icai from Pandey's Book in which $A(R_i) = R_f + \beta_1 \cdot Sh_1 + \beta_2 \cdot Sh_2 + \cdots + \beta_k \cdot Sh_k + e_i$
There is no such eqn in this universe but plz follow if Q. comes in exam....

Q. 49	pg
CW. 62	

Return = $R_f + \beta_1 Sh_1 + \beta_2 Sh_2 + \beta_3 Sh_3 + \beta_4 Sh_4 + \beta_5 Sh_5 + e$
error term (e) = 0

Factor	β	Sh = Actual - Expected	β×Sh
GNP	1.2	0	0
Inflation	1.75	1.5	2.625
Int. rate	1.30	1.25	1.625
Index	1.70	2	3.4
I. prodn	1.00	0.5	0.5
			8.15

Return = 9.25 + 8.15 + 0
= 17.40%

Self practice : Extra Q. 4) HW. Soln. 135

Arbitrage Pricing Theory :-

- Based on law of one price.
- Assets with identical risk must provide identical returns.
- If this law is violated → arbitrage opportunity.
- No Investment
- No Risk
- Positive Return

→ Type 1 sum : Informal - Typical (same hi ayega) (RTP).

Q.48	Pg
CW. 61	

Since Puma softech has positive return under all economic scenarios, arbitrage portfolio involves going long Puma & short on Him & Kalahari.

Given the prices, we will short sell 2 shares of Him & Kalahari each & buy 1 share of Puma.
Hence, initial investment = 0

Payoff under different scenarios -

① Recession -

Profit from Him	=	12% of 12 × 2	= 2.88
Loss from Kalahari	=	20% of 18 × 2	= (7.20)
Profit from Puma	=	18% of 60 × 1	= 10.80
		Payoff	= 6.48

② Moderate Growth -

Loss from Him	=	15% of 12 × 2	= (3.6)
Loss from Kalahari	=	12% of 18 × 2	= (4.32)
Profit from Puma	=	20% of 60 × 1	= 12
		Payoff	= 4.08

③ Boom -

Loss from Him	=	35% of 12 × 2	=	(8.4)
Profit from Kalhari	=	5% of 18 × 2	=	1.8
Profit from Puma	=	15% of 60 × 1	=	9.0
		Payoff	=	2.4

Conclusion :

Since payoff is positive under all scenarios, this is indeed an arbitrage portfolio.

Self practice : Extra Q. 3] HW. Soln. pg. 133

Self Note : Hint for payoff calculated above -
Humne Him aur Kalahari ko short sell kiya tha.... Aage jak buy krna pdega .... price jtni kam hogi utna badhiya hoga .... price kam mtlb return negative.... i.e. return negative toh profit & positive toh loss.
Similarly, Humne Puma buy kiya tha.... price jtni badhegi utha profit q k aage jak hum use sell krenge. price badhegi toh positive return ∴ Profit & vice-versa..

Part-2

→ Type 2 Sum : Formal

Based on Law of one price.

Eg : One factor APT

$R_e = 6 + 5\beta$

Stocks	Beta	E(R)
A	2	16%
B	2	14%

Eqn of no use

Conduct arbitrage.

Soln: Law of one price is violated.
Even though both stocks have the same Beta, they have different E(R).
Since, $E(R_A) > E(R_B)$, there is an arbitrage opportunity by long on A & short on B.

- Net Investment = 0
- Net Beta = 0

- $E(R) = 16 - 14 = 2\%$

Eg : One factor APT

$R_e = 6 + 5\beta$

Stocks	x	y	z
Beta	2.4	1.2	0.6
E(R)	18	12	11

Show the process of arbitrage.

Soln: Let us construct a hypothetical portfolio (P) comprising x & z such that β is equal to that of y i.e. 1.2

sabse jada β aur sabse kam β wale stocks ka portfolio banao = β of bich wala stock

$w_x \times 2.4 + w_z \times 0.6 = 1.2$

$w_x \times 2.4 + (1 - w_x) \times 0.6 = 1.2$

$\therefore w_x = 1/3$

$\& \ w_z = 2/3$

E(R) of portfolio $P = w_x E(R_x) + w_z E(R_z)$

$= 1/3 \times 18 + 2/3 \times 11$

$= 6 + 7.33$

$= 13.33\%$

Lets now compare y with P;

Stock / Portfolio	Beta	E(R)
y	1.2	12%
P	1.2	13.33%

So, law of one price is violated.

Arbitrage involves going long on P & short on y.

- Net Investment = 0
- Net β = 0
- $E(R) = 13.33 - 12 = 1.33\%$

Self practice: Extra Q.5] HW. Soln. 135